THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT 5120 Topics in Geometry 2021-22
Lecture 11 practice problems
2nd April 2022

- The practice problems are meant as exercise to the students. You are **NOT** required to submit your solutions, but you are encouraged to work through all of them in order to understand the course materials. The problems will be uploaded on Fridays and solutions will be uploaded on Wednesdays before the next lecture.

- Please send an email to **echlam@math.cuhk.edu.hk** if you have any questions.

1. Try to formulate and prove a hyperbolic version of Pythagorean theorem, based on what you know about the hyperbolic sine and cosine rules.

2. Let T be an equilateral hyperbolic triangle, say all of its side lengths are a, show that all three interior angles are equal. Say the interior angles are A, derive the relation

$$2 \cosh \frac{a}{2} \sin \frac{A}{2} = 1.$$

3. Let T be a right-angled hyperbolic triangle, say $A = \pi/2$, and suppose D is its hyperbolic area. Prove that
$$\sin D = \frac{\sinh b \sinh c}{1 + \cosh a}.$$

4. Let D_R be the hyperbolic disc of radius of radius R, and C_R be the hyperbolic circle bounding the disc, describe the behaviour of

$$q_{\mathbb{D}}(R) = \frac{\text{Length}(C_R)}{\text{Area}(D_R)}$$

when R becomes large. Compare this ratio to the one in Euclidean geometry.